Analysis of Fracture Design in a Vertical Well with a Horizontal Fracture

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Abstract
The overwhelming majority of hydraulic fractures are vertical and methods for designing them in both vertical and horizontal wells are well understood. In shallow and overpressured formations there is a strong likelihood for horizontal fractures. Hence, horizontal fractures are common in shallow coalbed methane formations and may occur in highly overpressured shale formations. Models for the transient flow and pressure behavior of horizontal fractures emanating from vertical wells exist and show highly distinct behavior from those for vertical fractures. Consequently, fracture design based on maximizing well productivity for a horizontally fractured well is distinct from the unified fracture design (UFD) strategies described previously for vertically-fractured vertical or horizontal wells.

This study shows well productivity behavior for horizontal fractures in homogeneous formations with or without vertical to horizontal permeability anisotropy as a function of suitably-defined proppant number, dimensionless fracture conductivity, and fracture penetration index parameters. For low penetration index, a relationship among fracture half-length, conductivity, and skin is provided.

This work provides a simple framework for horizontal fracture design.

Introduction
Hydraulic fracturing was first reported in the early 1950s (Grossman, 1951) and has been extensively used to improve the well productivity in the oil and gas industry. Different fracturing techniques have been developed in the past 60 years, but the overwhelming majority of hydraulic fractures are vertical in the field practice, and the methods for designing them in both vertical and horizontal wells have been widely studied and well understood. The idea of horizontal fractures, however, does not receive much attention, and a limited volume of literature can be found about it in contrast to vertical fractures.

The application of horizontal fractures was greatly restricted because it was generally believed that in conventional reservoirs the vertical permeability is much lower than the horizontal (usually one tenth of the horizontal permeability), so that vertical fractures will be more beneficial than horizontal ones. Landrum and Crawford (1957) also found that horizontal fractures are not very effective in thick pays and if the well is not centered in the drainage area. A third reason is the old “common wisdom” (Gidley, et al, 1989) that horizontal fractures cannot occur below 2,000 feet since the maximum stress direction is usually vertical, but Wright et al. (Wright et al, 1997) challenged it and reported a case of horizontal fractures at the depth of 7,500 feet due to high horizontal reservoir stress. There is another technical issue (Lowe and Huit, 1966) that proppants may deposit and form a dune around the wellbore in the horizontal fracture at high proppant concentration or low fluid velocity during the fracturing process, which is not likely for vertical fractures due to gravity. The dune will block the whole flow path and reduce the fracture flow capacity, resulting in shorter fracture length than designed.

Despite of these shortcomings of horizontal fractures, it has been recognized nowadays that horizontal fractures are far more common than generally believed. In fact, it was reported as early as in 1960 (Morrisson and Henderson, 1960) that horizontal fractures were created to produce gravity drainage reservoirs. Horizontal fractures are also applicable to thin pays because of their low cost even though it has been shown by Sung et al. (Sung and Ertekin, 1987) that horizontal wells have better performance. Moreover, horizontal fractures are common in unconventional reservoirs like shallow coal bed methane or heavy oil formations and highly over-pressured shale formations (Valko and Economides, 1998; Chhina et al, 1987; Britt et al, 2006). In addition, in-situ stress state change due to geological movements (e.g. a thrust fault) (Maxwell et al, 2007), secondary or enhanced oil recovery (Wright et al, 1997; Wong and Chau, 2004) may result in significant growth of horizontal fractures during fracturing.

Daal and Economides (2006) studied hydraulically fractured wells in square and rectangular patterns, and they concluded that a dimensionless fracture conductivity of 1.6 will yield the highest productivity for production systems with proppant number below 0.1. Recently, Larsen (2011) did a type curve study of horizontal fractured wells in single and multilayer reservoirs. Their work brought up our interest in horizontal fractures and initialized this paper. We wanted to provide a simple framework for horizontal fracture design. And in our study we investigated well productivity behavior for horizontal fractures in homogeneous formations with or without vertical to horizontal permeability anisotropy as a function of suitably-defined proppant number, dimensionless fracture conductivity, and fracture penetration index parameters. We used the software Eclipse to build our models and run the simulation.

Upon completion of our simulation using our Eclipse model, we found that in a circular reservoir, the maximum $C_{FD}$ is no longer 1.6 but changes even at low proppant number below 0.1. The maximum $J_D$ is also seen to be greater than 6/$\pi$ as seen in a square or rectangular reservoir.
Upon doing sensitivity analysis, we also found that changing the effective radius increases the dimensionless productivity index at a constant value of dimensionless conductivity. Also, we found that by changing Kv/Kh, we are exponentially increasing the dimensionless production index.

**Model Description**

This section will introduce the model used in this work, its validity for this work, and data treatment after the simulation. Reservoir simulator Eclipse was used to build the model. After the model was set up, we run some known cases from other papers to verify our model. Finally, the calculation based on the simulated data was introduced.

**Model for Simulation.** The model used is a radial reservoir, with a well fully penetrating the formation right in the center. The fracture is created as radial fracture, with the radius of \( r_f \). This model is shown in **Figure 1**.

![Figure 1—Reservoir model with fracture.](image)

In **Figure 1**, different colors represent different layers. In this model, we divide the formation into 11 layers, with one layer containing the horizontal fracture. The horizontal fracture is contained in the middle layer. Note that only 10 layers can be observed in Figure 1 as the middle layer is too thin to be observed. If we make the middle layer thicker, shown in **Figure 2**, the layer with fracture can be seen, but this is just for illustration. In the model calculation, the middle layer is assigned to a very small thickness.

Radial coordinated was used for this theoretical model. The radial direction is discretized into 50 grids, theta direction is treated as one grid, and vertical direction is discretized into 11 even grids. The algorithm for discretizing radius is **Eq. 1**:

\[
\frac{r_{i-1}}{r_i} = \left( \frac{r_e}{r_{i-1}} \right)^{\frac{n-1}{n}} \quad \text{..................................................(1)}
\]

In this equation, \( i \) is the grid number in radial direction while \( n \) is the total grid number in radial direction, which is 50 in this case.

The fracture is treated by changing the layer thickness and grid permeability and porosity. The desired grid permeability can be calculated by the **Eq. 2** which provides the proppant number and reservoir properties.

![Figure 2—Reservoir model with visible fracture in the middle layer.](image)

\[
N_{p-HF} = \frac{k_{f w r} t}{k_f r_f^2} \quad \text{..................................................(2)}
\]

Note that in our model, the fracture width is not calculated but selected by us. As the fracture permeability need to be set a reasonable value, neither too large nor too small, to avoid simulation failure. And we can see, inside the fracture, the permeability is different in different radial grids.

In our simulator, we assigned the whole layer with the same thickness as fracture width. The fracture may not necessarily penetrate fully the whole formation, so that each layer has two different permeability and porosity, with inner part for the fracture and outer part for the reservoir.

**Model Verification.** Since we treat the hydraulic fracture as formation with high permeability, porosity and tiny width, we need to verify whether this method is appropriate or not. This part provides the simulated results for a horizontal fracture in a large reservoir. The input data is from Larsen (2011).

After the simulation, the flowing bottom-hole-pressure with time can be exported. Thus a log-log diagnostic plot can be generated as shown in **Figure 3**. This graph is consistent with the graph in Larsen’s paper (2011). In conclusion, the model can be used to simulate the horizontal fracture.

**Calculation Based on the Simulated Data.** The methods for calculating the dimensionless productivity index and dimensionless horizontal fracture conductivity are illustrated in this part.

The dimensionless horizontal fracture conductivity measures the relative ease of the fluid flowing from reservoir to fracture and then to the wellbore. It can be calculated as **Eq.3**.
the problem solution.

Case Analysis. There are nine different cases for analysis using the reservoir model designed in Eclipse as shown in Table 2.

### Table 1—Reservoir Properties for Model Analysis

| Initial pressure, \( p_i \) (psi) | 3,000 |
| Wellbore radius, \( r_w \) (ft) | 0.3 |
| Porosity, \( \phi \) | 0.1 |
| Fluid viscosity, \( \mu \) (cp) | 1 |
| Total compressibility, \( c_t \) (1/psi) | 13.10*10^-6 |
| Fracture number, \( n_f \) | 1 |
| Fracture half length, \( x_f \) (ft) | 200 |
| Skin, \( s \) | 0 |

## Results Analysis

In order to achieve our goal of analyzing horizontal fractures in homogenous formations with a vertical well as a function of proppant number, dimensionless fracture conductivity, and fracture penetration index parameters, Eclipse was used to design the model. The following sections will include the actual problem statement, reservoir properties of the formation which are the inputs of our Eclipse model. And we will take an in-depth look at nine different cases of \( J_D \) versus \( C_{FD-HF} \) with changes in \( N_{p-HF} \), \( x_s/h \), and also differences in isotropic and anisotropic reservoirs.

**Problem Statement.** Given a specific set of reservoir properties for designing a vertical well with horizontal fractures, we analyzed the scenario of JD versus \( C_{FD-HF} \). Table 1 describes the reservoir properties we used to design
sense because as we have more effective radius holding the permeability constant, we will have more production due to the increase drainage area.

The increase in $k_v/k_h$ versus $J_D$ is an exponential increase. This is important because in designing a reservoir to maximize $J_D$ to minimize proppant cost, we want to have a higher $k_v/k_h$ ratio while increasing the dimensionless conductivity. This is shown in Figure 6, a sensitivity analysis of $k_v/k_h$ by a power of 10.

The consistency with all case scenarios shown in the Appendix (Figures 7 to 14) also proves the importance of designing with the optimum $k_v/k_h$ ratio in mind as it has the greatest effect over the productivity index. Knowing the optimum dimensionless conductivity ($C_{ID}$) also makes the hydraulic fracture stimulation much more cost-effective to apply to enhance production because of the reduction in unnecessary use of proppant.

Conclusions
1. At a higher proppant number we will see a higher dimensionless productivity index at every value of conductivity
2. In order to maximize productivity we need maximum the effective drainage radius.
3. We want a higher $k_v/k_h$ ratio when producing in a vertical well with a horizontal fracture to minimize cost of proppant.

Nomenclature

- $r_i$ = radial position of $i$-th grid, ft
- $r_e$ = reservoir radius, ft
- $r_f$ = fracture radius, ft
- $k_f$ = fracture permeability, md
- $k_v$ = formation vertical permeability, md
- $k_h$ = formation horizontal permeability, md
- $N_{p, HF}$ = horizontal fracture proppant number
- $C_{ID, HF}$ = dimensionless horizontal fracture conductivity
- $J$ = well productivity index, STB/D/psi
- $J_D$ = dimensionless well productivity index
- $Q$ = production rate, STB/D
- $P_{ave}$ = average reservoir pressure, psi
- $P_{wf}$ = bottom hole flowing pressure, psi
- $B$ = formation volume factor, rb/STB
- $h$ = formation height, ft
- $w$ = fracture width, ft

Greek variables

- $\mu$ = fluid viscosity, cp

Subscripts

- $f$ = fracture
- HF = horizontal fracture

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References
Figure 11—Case 5 with $x_e/h = 20$ and $k_v/k_h = 1$.

Figure 12—Case 6 with $x_e/h = 50$ and $k_v/k_h = 1$.

Figure 13—Case 8 with $x_e/h = 20$ and $k_v/k_h = 0.01$.

Figure 14—Case 9 with $x_e/h = 50$ and $k_v/k_h = 0.01$.